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Symmetries in Hall-like systems: microwave and nonlinear transport effects

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Abstract

In this work, we present a model to describe the nonlinear response to a dc electrical current of a two-dimensional electron system subjected to magnetic and microwave fields. Considering the separation of the electron coordinates into the non-commuting relative and guiding center coordinates, we obtain a unitary transformation that exactly solves the time-dependent Schrödinger equation in the presence of arbitrarily strong electric, magnetic and microwave fields. Based on this formalism, we provide a Kubo-like formula that takes into account the oscillatory Floquet structure of the problem. We discuss results related to the recently discovered zero-resistance states and to the microwave-induced resistivity oscillations and the Hall-induced resistivity oscillations.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Non-equilibrium magnetotransport in high mobility two-dimensional electron systems (2DES) has acquired great experimental and theoretical interest. Recently, two experimental groups [1–4] have reported the unexpected discovery of zero-resistance states (ZRS) when high mobility $GaAs/Al_xGa_{1-x}$ As heterostructures in weak magnetic fields were exposed to millimeter irradiation. Unlike the strong magnetic field regime, the Hall resistance is not quantized, but the magnetoresistance exhibits strong oscillations. These microwave-induced oscillations (MIRO) are periodic in $\epsilon^{\rm ac} = \omega/\omega_c$, where ω and ω_c are the microwave and cyclotron frequencies respectively. The series of minima formed at $\epsilon^{\rm ac} = j + \delta^{\rm ac}$, with $j = 1, 2, 3....$, and $\delta^{\rm ac} < 1/2$ depends on the parameters of the samples: in the experiments of Zudov *et al* [1, 2] it was found $\delta^{\rm ac} \approx \frac{1}{2}$ whereas as for Mani *et al* [3, 4], $\delta^{\rm ac} \approx \frac{1}{4}$. Our current understanding of this phenomenon rests upon models that predict the existence of negative resistance (NRS). It was argued that negative resistance induces the formation of current domains, yielding an instability that drives the system into a ZRS [5]. Nowadays two distinct mechanisms that produce negative longitudinal conductance are known: (i) *the impurity scattering mechanism*, which is caused by the disorder-assisted absorption and emission of microwaves [6–14], and (ii) *the distribution function mechanism*, according to which the microwave absorption modifies the electron distribution function leading to a negative longitudinal conductance [15–18]. Similar to MIRO, there are magnetoresistance oscillations induced by the combined effects of microwave irradiation and periodic potential modulation [19–21].

More recently, Hall-field-induced resistance oscillations (HIRO) have been observed in high mobility samples in response to a strong dc electric current [22, 23]. The HIRO oscillations are periodic in the inverse magnetic field, with the resistance maxima appearing at integer values of the dimensionless parameter $\epsilon^{\mathrm{dc}} = \omega_H/\omega_c$. The Hall frequency $\omega_H = \gamma\,(2\pi/n_e)^{1/2}\,\mathcal{J}_x/e$ is associated with the energy $\hbar\omega_H = \gamma R_c E_y^{cl}$, where the classical Hall electric field is given by $E_y^{cl} = B\mathcal{J}_x/en_e$, R_c is the cyclotron radius of the electrons at the Fermi level and $\gamma \sim 1.63$–2.18. These results were confirmed in the recent experiment by Zhang *et al* [24], with a determination of the parameter $\gamma \sim 1.9$. Additionally in this work another notable nonlinear effect was found: in the regime of separated LLs a relative weak dc current induces a dramatic reduction of the resistivity. Although MIRO and HIRO are basically different phenomena, both rely on the commensurability of the cyclotron frequency with a characteristic parameter, ω and ω_H respectively. The study of HIRO permits us to analyze resistivity as a function of ϵ^{dc} when both ω_c and ω_H are varied over a wide range of frequencies. On the other hand, the MIRO studies are carried out performing ω_c-sweeps at fixed ω, because of the experimental difficulty in implementing ω-sweeps. Other interesting examples of nonlinear magnetotransport experiments combine the microwave and the dc-current excitations [25].

Some theoretical studies of the nonlinear transport properties under dc excitations have recently appeared [26–28]. The aim of this work is to develop a model to describe both the microwave and the nonlinear response to a dc electrical current of a 2DES placed on a magnetic field. Considering the separation of the electron coordinates into its relative $\mathbf{R}$ and guiding center $\mathbf{X}$ coordinates, we obtain a unitary transformation that exactly solves the time-dependent Schrödinger equation in the presence of arbitrarily strong electric, magnetic and microwave fields. Although the relative and guiding center coordinates commute the R_x and R_y relative coordinates satisfy a non-commutative algebra (and similarly for the X and Y projections of the relative coordinates). These properties are exploited to work out a Kubo-like formula for the conductivity as a response to randomly distributed impurities. Considering the case of weak dc excitation, we recover the linear response result with respect to the electric field intensity. The MIRO can be studied in this limit. In the general nonlinear regime, self-consistent expressions determine the longitudinal and Hall components of the electric field in terms of the dc current density $\mathcal{J}_x$. The differential resistance displays a strong oscillatory behavior resulting from current-assisted electron scattering producing both intra-LL and inter-LL transitions. As an application of our model we discuss results related to the negative-resistance states and to the microwave-induced resistivity oscillations (MIRO) and the Hall-induced resistivity oscillations (HIRO).

2. Model

We consider the nonlinear response to a dc electrical current density ($\mathcal{J}_x$) of a 2DES placed in a magnetic field and irradiated by microwaves. The electrons are subjected to: (1) a magnetic $\mathbf{B} = (0, 0, B)$, (2) an in-plane electric $\mathbf{E} = E(\cos\theta, \sin\theta, 0)$ and (3) microwave fields. Additionally, the effects of the impurity scattering potential V have to be included; hence the dynamics is governed by the Hamiltonian

$$H = H_{\{B,E,\omega\}} + V, \tag{1}$$

where

$$H_{\{B,E,\omega\}} = \frac{1}{2m^*}\Pi^2 + e\mathbf{E}_{\mathrm{dc}} \cdot \mathbf{x}, \tag{2}$$

here m^* is the effective electron mass. $H_{\{B,E,\omega\}}$ includes the interaction with the dc electric fields, and the interaction with the magnetic and microwave fields via the covariant momentum $\Pi = -\mathrm{i}\hbar\nabla + e\mathbf{A}$; with the vector potential selected as

$$\mathbf{A} = -\frac{1}{2}\mathbf{r} \times \mathbf{B} + \mathrm{Re}\left[\frac{\mathbf{E}_\omega}{\omega}\exp\{-\mathrm{i}\omega t\}\right]. \tag{3}$$

The impurity potential is decomposed in terms of its Fourier components

$$V(\mathbf{r}) = \sum_i^{N_i} \int \frac{\mathrm{d}^2 q}{(2\pi)^2} V(q) \exp[\mathrm{i}\mathbf{q} \cdot (\mathbf{r} - \mathbf{r}_i)], \tag{4}$$

where $\mathbf{r}_i$ is the position of the ith impurity and N_i is the number of impurities. The explicit form of $V(q)$ depends on the nature of the scatterers. For short-range neutral impurities [12]: $V(q) = 2\pi\hbar^2\alpha/m^*$, where α is the scattering length, and the impurity density is related to the electron mobility according to the relation $\alpha^2 n_{\mathrm{imp}} = e/(4\pi^2\hbar\mu)$. Instead, in the case of a 2D screened Coulomb potential: $V(q) = \frac{\pi\hbar^2 q_{TF}}{m^*}\,\mathrm{e}^{-qd}/(q_{TF} + q)$, where d is the thickness of the doped layer and $q_{TF} = e^2 m^*/(2\pi\epsilon_0\epsilon_b\hbar^2)$. In this case the relation of the impurity density to the electron mobility is approximated as $n_{\mathrm{imp}} = 8e(k_F d)^3/(\pi\hbar\mu)$. Here we shall only consider the case of neutral impurities.

A planar electron performs cyclotron and drifting motion in magnetic and electric fields. It is then convenient to decompose the electron coordinate $\mathbf{r}$ into the guiding center $\mathbf{X} = (X, Y)$, and the relative coordinate $\mathbf{R} = (R_x, R_y)$, i.e. $\mathbf{r} = \mathbf{X} + \mathbf{R}$, where $\mathbf{R} = (-\Pi_y/eB, \Pi_x/eB)$. The commutation relations are

$$[R_x, R_y] = \frac{\mathrm{i}\hbar}{eB}, \qquad [X, Y] = -\mathrm{i}l_B^2, \qquad [X_i, R_j] = 0, \tag{5}$$

with $l_B^2 = \hbar/eB$. With this decomposition the X and Y coordinates become noncommutative (similarly for R_x and R_y). The mathematical analysis of this kind of non-commutative structures leads to interesting mathematical developments related to the exotic Galilei group [29, 30].

Ignoring for a moment the impurity potential, it is possible to find a unitary transformation $\Psi^{(W)} = W\Psi$ that exactly solves the time-dependent Schrödinger equation: here $\Psi^{(W)}$ is the solution of

$$\hbar\omega_c\left(\tfrac{1}{2} + a^\dagger a\right)\Psi^{(W)}_{\mu,k} = \mathcal{E}_{\mu,k}\Psi^{(W)}_{\mu,k}, \tag{6}$$

μ labels the Landau levels and k is the eigenvalue of the transverse $-X\sin\theta + Y\cos\theta$ center of guide coordinate, and the ladder operator a is given by

$$a = \frac{1}{\sqrt{2eB}}\left(\Pi_x - \mathrm{i}\Pi_y\right). \tag{7}$$

The unitary transformation is given by

$$W(t) = \exp\left\{\frac{\mathrm{i}}{\hbar}m\dot{\xi} \cdot \mathbf{X}\right\}\exp\left\{\frac{\mathrm{i}}{\hbar}(m\dot{\xi} - e\xi \times \mathbf{B}) \cdot \mathbf{R}\right\}\exp\left\{\mathrm{i}\int^t \mathcal{L}\,\mathrm{d}t'\right\}, \tag{8}$$

where $\mathcal{L}$ is the classical Lagrangian

$$\mathcal{L} = \tfrac{1}{2} m\dot{\xi}^2 + e\dot{\xi}\cdot A + e\mathbf{E}\cdot\xi, \tag{9}$$

and $\xi(t)$ solves the corresponding equations of motion. The energy eigenvalues in equation (6) are readily determined as

$$\mathcal{E}_\mu = \hbar\omega_c\left(\tfrac{1}{2}+\mu\right) + \mathcal{E}_{\rm rad} + ekE_{\rm dc}, \qquad \mathrm{Mod}[\hbar\omega], \tag{10}$$

with

$$\mathcal{E}_{\rm rad} = \frac{e^2E_\omega^2[1+2\omega_c \mathrm{Re}(\epsilon_x^*\epsilon_y)/\omega]}{2m^*[(\omega-\omega_c)^2+\Gamma^2]}, \tag{11}$$

here the microwave electric field is decomposed as $\mathbf{E}_\omega = E_\omega(\epsilon_x,\epsilon_y)$.

We now turn to the calculation of the current density. The velocities of the center guide coordinates are obtained from the Heisenberg operator equations using the total Hamiltonian in equation (1):

$$\begin{aligned}\dot{X} &= \frac{\mathrm{i}}{\hbar}[H,X] = \frac{E_y}{B} - \frac{l_B^2}{\hbar}\frac{\partial V}{\partial y},\\ \dot{Y} &= \frac{\mathrm{i}}{\hbar}[H,Y] = -\frac{E_x}{B} + \frac{l_B^2}{\hbar}\frac{\partial V}{\partial x}.\end{aligned} \tag{12}$$

The current density is computed from the impurity and thermal average $\mathcal{J} = e\langle \mathrm{Tr}[\rho\,(t)\,\mathbf{V}]\rangle$ of the center of guide velocity $\mathbf{V} = (\dot{X},\dot{Y})$, weighted with the density matrix that satisfies the von Neumann equation

$$\mathrm{i}\frac{\partial}{\partial t}\rho = [H_0+V,\rho]. \tag{13}$$

We shall now derive the Kubo–Greenwood formula for the current within the framework of the linear response theory with respect to the impurity potential. The external field effects are exactly taken into account through the exact wave function solution given. The Hamiltonian is split into an unperturbed part $H_{\{B,E\omega\}}$ and the perturbation $V(\mathbf{r})\exp(-\delta|t|)$; note that we added to the impurity potential a term $\exp(-\delta|t|)$, with δ representing the rate at which the perturbation is turned on and off. The density is similarly decomposed as $\rho = \rho_0 + \Delta\rho$, where the unperturbed density matrix takes the form $\rho_0 = \sum_\alpha f(\mathcal{E}_\alpha)|\alpha\rangle\langle\alpha|$; where f is the Fermi distribution function. Following the usual procedure of the linear response formalism, the matrix elements of $\Delta\rho$, in the base given by states $\Psi_{\mu,k}^{(W)}$, are worked out as

$$\langle\mu,k|\Delta\rho|\nu,k'\rangle = \langle\mu,k|V|\nu,k'\rangle(f_{\nu,k'}-f_{\mu,k})\frac{1}{\mathcal{E}_{\nu,k'}-\mathcal{E}_{\mu,k}+\mathrm{i}\delta}. \tag{14}$$

Combining the previous equations, and assuming randomly distributed impurities, the average current density is explicitly computed; as suggested by equation (12) the current is split into drifting and impurity scattering contributions:

$$\mathcal{J}_i(\vec{E}) = \epsilon_{ij}\frac{en_e}{B}E_j + \mathcal{J}_i^{(\rm rad)}(\vec{E}), \tag{15}$$

with the impurity contribution given by

$$\begin{aligned}\mathcal{J}_i^{(\rm rad)}(\vec{E}) = -\frac{en_I}{\hbar}\epsilon_{ij}\sum_{\mu\nu}\sum_l\int\frac{\mathrm{d}^2q}{(2\pi)^2}q_j|J_l(|\Delta|)V_{\rm imp}(\vec{q})D_{\mu\nu}(\tilde{q})|^2&\\ \times[f(\mathcal{E}_\mu+\omega_q+l\omega)-f(\mathcal{E}_\nu)]\rho_\nu(\mathcal{E}_\mu+l\omega+\omega_q).&\end{aligned} \tag{16}$$

$\mathcal{J}_i^{(\rm rad)}$ is evaluated at arbitrary values of the electric field through the argument dependence of

$$\omega_q = el_B^2(q_yE_x - q_xE_y). \tag{17}$$

In equation (16) J_l denotes the Legendre polynomials and its argument Δ arises from the solution of the classical equations of motion for ξ and it is given by

$$\Delta = \frac{\omega_c l_B^2 e E_\omega}{\omega(\omega^2 - \omega_c^2 + \mathrm{i}\omega\Gamma)}[\omega(q_x\epsilon_x + q_y\epsilon_y) + \mathrm{i}\omega_c(q_x\epsilon_y - q_y\epsilon_x)]. \tag{18}$$

The matrix elements $D_{\mu,\nu}$ in equation (16) are given by

$$D^{\nu\mu}(\tilde q) = \langle \nu\,|D(\tilde q)|\,\mu\rangle = \mathrm{e}^{-\frac{1}{2}|\tilde q|^2} \begin{cases} (-\tilde q^*)^{\mu-\nu}\sqrt{\dfrac{\nu!}{\mu!}}L_\nu^{\mu-\nu}(|\tilde q|^2), & \mu > \nu, \\ \tilde q^{\nu-\mu}\sqrt{\dfrac{\mu!}{\nu!}}L_\mu^{\nu-\mu}(|\tilde q|^2), & \mu < \nu, \end{cases} \tag{19}$$

with $\tilde q = (q_x - \mathrm{i}q_y)/\sqrt{2}$ and $L_\nu^{\mu-\nu}(|\tilde q|^2)$ denotes the associated Laguerre polynomial.

For the density of states a Gaussian-type expression is justified [32–34]:

$$\rho_\nu(\omega_{\mathbf q} + E_\nu - E_\mu) = \sqrt{\frac{\pi}{2\Gamma^2}}\exp\left[-\frac{(\omega_{\mathbf q} + E_\nu - E_\mu)^2}{2\Gamma^2}\right], \qquad \Gamma^2 = \frac{2\hbar^2\omega_c}{\tau_s}. \tag{20}$$

The single-particle scattering time τ_s is related to the transport scattering time τ_{tr} obtained from the mobility according to the relation $\tau_s = \tau_{\mathrm{tr}}/\beta$. In the case of short-range scatterers $\tau_{\mathrm{tr}} = \tau_s$ and $\beta = 1$. In the case of long-range screened potential, β_ν depends on the filling factor ν, e.g. $\beta_{\nu=50} \approx 13.5$ [12].

The present formalism assumes that inelastic scattering processes can be neglected; this approximation is expected to be valid for a sufficiently small temperature. If the inelastic relaxation time τ_i fulfils the condition $\tau_{\mathrm{tr}} \ll \tau_i$, then inelastic processes will not contribute to the evaluation of the electron mobility. τ_{tr} can be estimated from $\mu = e\tau_{\mathrm{tr}}/m^*$ as $\tau_{\mathrm{tr}} \sim 0.9$ ns. To get an order of magnitude estimation, let us consider inelastic phonon scattering; an approximated expression for τ_i is given by [31]

$$\frac{1}{\tau_i} = \frac{3m^* b k_B T\,\Xi^2}{16\rho v_s^2\hbar^3}. \tag{21}$$

Typical values are $\rho v_s^2 = 1.4\times 10^{11}$ J m^{-3}, deformation potential $\Xi = 3$ eV, Fang–Howard parameter $b \approx 0.2$ nm^{-1}. The condition to neglect inelastic phonon processes is estimated as $T < 20$ K. A detailed calculation for the electron scattering processes to τ_i has been carried out in [17]; the results show that τ_i has a T^{-2} dependence, obtaining for $T \sim 1$ K, $\tau_i \sim 4$ ns, just slightly larger than τ_{tr}. It will be interesting to extend the present formalism to include inelastic processes; this can be implemented if one adds the electron–phonon and electron–electron potential interactions to the potential V in equation (1). At this stage one should not ignore that there is an ongoing controversy in relation to the physical origin of the MIRO and HIRO. (i) The impurity scattering mechanism [6–14] and (ii) the mechanism based on the distribution function modified by inelastic processes [15–18] have been used to analyze both the MIRO and HIRO; but it will be the case that the two processes apply in different regimes, or that they play a complementary role [25].

The current in equation (15) applies, in general, to the nonlinear transport regime for an arbitrary strength of the dc electric field. The linear response regime is recovered expanding to first order in E_x (the zero-order term cancels because of the angular integration) to yield: $J_x = \sigma_{xx}E_x$ and $J_y = \sigma_{yx}E_x$, with $\sigma_{xy} = \sigma_{yx} = en/B$ and

$$\sigma_{xx} = -\frac{en_I}{\hbar}\sum_{\mu\nu}\sum_l \int \frac{\mathrm{d}^2 q}{(2\pi)^2} q_y^2 |J_l(|\Delta|)V_{\mathrm{imp}}(\vec q)D_{\mu\nu}(\vec q)|^2 \\ \times [f(\mathcal{E}_\mu + l\omega) - f(\mathcal{E}_\nu)]\rho_\nu(\mathcal{E}_\mu + l\omega). \tag{22}$$

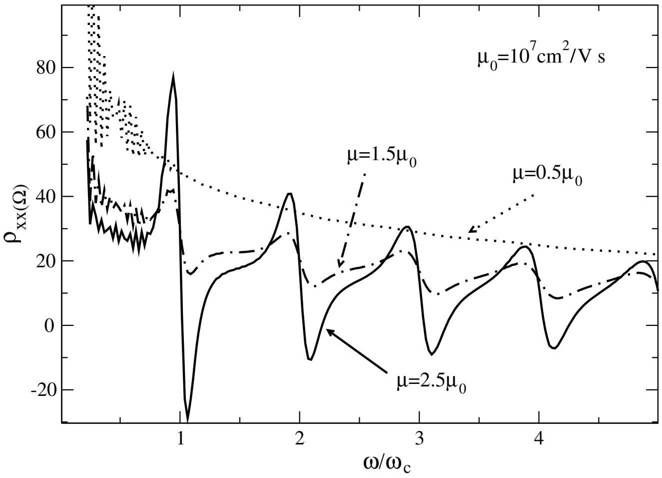

Figure 1. Longitudinal resistivity in the linear response regime ($\mathcal{J}_x \ll 1$) as a function of $\epsilon^{\mathrm{ac}} = \omega/\omega_c$ for three values of the electron mobility: $\mu \approx 0.5 \times 10^7$ cm^2 V^{-1} s^{-1} (dotted line), $\mu \approx 1.5 \times 10^7$ cm^2 V^{-1} s^{-1} (dash-dotted line), and $\mu \approx 2.5 \times 10^7$ cm^2 V^{-1} s^{-1} (continuous line). In the two former cases the oscillations follow a pattern with minima at $\epsilon^{\mathrm{ac}} = j + \delta^{\mathrm{ac}}$, and maxima at $\epsilon^{\mathrm{ac}} = j - \delta^{\mathrm{ac}}$, adjusted with $\delta^{\mathrm{ac}} \approx 1/5$. NRS only appear when $\mu > \mu_{\mathrm{th}} \sim 1.5 \times 10^7$ cm^2 V^{-1} s^{-1}. The selected parameters are: $m^* = 0.0635 m_e$, $T \approx 1$ K, $f = 100$ GHz, $|\vec{E}_\omega| \approx 2.5$ V cm^{-1}, $n_I = 1.5 \times 10^{11}$ cm^{-2}, and $n_e = 3.7 \times 10^{11}$ cm^{-2}.

The corresponding resistivities are obtained from the expression $\rho_{xx} = \sigma_{xx}/\left(\sigma_{xx}^2 + \sigma_{xy}^2\right)$ and $\rho_{xy} = \sigma_{xy}/\left(\sigma_{xx}^2 + \sigma_{xy}^2\right)$. The relation $\sigma_{xy} \gg \sigma_{xx}$ holds in general, hence it follows that $\rho_{xx} \propto \sigma_{xx}$, and the longitudinal resistivity follows the same oscillation pattern as that of σ_{xx}.

We first consider an example of MIRO. The system is microwave irradiated and we assume the linear response limit with respect to the dc current; $\mathcal{J}_x \ll 1$ allow us to use equation (22). For the microwave ac-induced oscillations a convenient control parameter is the ratio of microwave to the cyclotron frequencies: $\epsilon^{\mathrm{ac}} = \omega/\omega_c$. Figure 1 displays ρ_{xx} versus ϵ^{ac} for three selected values of the electron mobility $\mu = e\tau_{\mathrm{tr}}/m^*$. For $\mu \approx 0.5 \times 10^7$ cm^2 V^{-1} s^{-1} and almost linear behavior $\rho_{xx} \propto B$ is clearly depicted. As the electron mobility increases to $\mu \approx 1.5 \times 10^7$ cm^2 V^{-1} s^{-1}, MIRO are clearly observed; a further increase to $\mu \approx 2.5 \times 10^7$ cm^2 V^{-1} s^{-1} leads to the appearance of negative resistance states. It is observed that ρ_{xx} vanishes at $\epsilon^{\mathrm{ac}} = j$ for j integer. The period and phase of the oscillations follow a pattern very similar to that observed in experiments [1, 3], with minima at $\epsilon^{\mathrm{ac}} = j + \delta^{\mathrm{ac}}$, and maxima at $\epsilon = j - \delta^{\mathrm{ac}}$, with $\delta^{\mathrm{ac}} \approx 1/5$. It should be pointed out that this value of δ^{ac} depends on the representation of the density of states and of the precise value of the width Γ. This fact may be the explanation for the different determination of δ^{ac} in experiments: Zudov *et al* [1, 2] found $\delta^{\mathrm{ac}} \approx \frac{1}{2}$ whereas as for Mani *et al* [3, 4], $\delta^{\mathrm{ac}} \approx \frac{1}{4}$.

Let us now consider the nonlinear transport regime. In a typical experimental configuration the electric field is not explicitly controlled, instead the longitudinal current $\mathcal{J}_x$ is fixed to a constant value, while the transverse Hall current $\mathcal{J}_y$ cancels. Consequently equations (15) lead to the conditions

$$\mathcal{J}_x = \frac{en_e}{B} E_y + \mathcal{J}_x^{(\mathrm{imp})}(E_x, E_y), \qquad 0 = -\frac{en_e}{B} E_x + \mathcal{J}_y^{(\mathrm{imp})}(E_x, E_y). \tag{23}$$

They represent two implicit equations for the unknowns E_x and E_y; the equations can be solved following a self-consistent iteration. However, it is easily verified that in general the following conditions apply: $E_x \ll E_y$ and $\mathcal{J}_x^{(\mathrm{imp})} \ll en_e E_y/B$. It is then possible to explicitly solve for E_x and E_y

$$E_y = \frac{B}{en_e}\mathcal{J}_x - \frac{B}{en_e}\mathcal{J}_x^{\mathrm{imp}}(E_x, E_y) \approx \frac{B}{en_e}\mathcal{J}_x - \frac{B}{en_e}E_x\left(\frac{\partial \mathcal{J}_x^{\mathrm{imp}}(E_x, E_y)}{\partial E_x}\right)_{(E_x=0, E_y=B\mathcal{J}_x/en_e)}$$
$$E_x = \frac{B}{en_e}\mathcal{J}_y^{\mathrm{imp}}(E_x, E_y) \approx \frac{B}{en_e}\mathcal{J}_y^{\mathrm{imp}}(E_x = 0, E_y = B\mathcal{J}_x/en_e). \tag{24}$$

In order to derive the previous results we used the fact that $\mathcal{J}_x^{\mathrm{imp}}(E_x = 0, E_y)$ cancels because of the angular integration. Equation (24) shows that the leading contribution to the Hall electric field is given by the classical result $E_y^{cl} = B\mathcal{J}_x/en_e$, and the Hall resistivity is given again by the expression $\rho_{xy} = B/en_e$. Instead, the expression for the nonlinear longitudinal resistivity is given as

$$\rho_{xx} = \frac{E_x}{\mathcal{J}_x} = \frac{B}{en_e\mathcal{J}_x}\mathcal{J}_y^{\mathrm{imp}}(E_x = 0, E_y = B\mathcal{J}_x/en_e). \tag{25}$$

Equations (15), (16) with the definitions in equations (17)–(20) apply in general to the nonlinear regime in which both the longitudinal and Hall electric fields are arbitrarily strong. However, for the conditions in experiments of current interest, it is reasonable to consider the E_x weak limit. Then, the Hall field is accurately approximated by the classical result $E_y = BJ_x/en_e$, whereas ρ_{xx} can be computed from equation (25).

In the work of Zhang *et al* [24] the Hall frequency is defined as $\omega_H = \gamma J_x(2\pi/e^2 n_e)^{1/2}$. Here we assume that $\gamma = 2$; this can be justified if we observe that the integral in equation (16) is evaluated in terms of the variable $\omega_q^* = q_x J_x/en_e$ and it is dominated by contributions of exchanged momentum in the region $q_x \approx 2k_F$. Recalling that $k_F = \sqrt{2\pi n_e}$, it yields $\omega_H = J_x(8\pi/e^2 n_e)^{1/2}$. It is convenient to define a dimensionless control parameter ϵ^{dc} that is given by the ratio of the Hall to the cyclotron frequencies

$$\epsilon^{\mathrm{dc}} = \frac{\omega_H}{\omega_c} = \frac{2eE_yR_c}{\hbar\omega_c}, \qquad R_c = \frac{v_F}{\omega_c}, \tag{26}$$

and it can be interpreted as the ratio of the work of the electric Hall field associated with the displacement of the guiding center of the cyclotron trajectory by $2R_c$ to the Landau energy $\hbar\omega_c$.

Next we consider the case of HIRO, the microwave radiation is switched-off, but the current density is strong enough to induce Hall-field-induced magneto oscillations. Figure 2(*a*) shows the differential resistance $r_{xx} = \partial(\mathcal{J}_x\rho_{xx})/\partial\mathcal{J}_x$ as a function of the magnetic field B for a fixed current density $J_x = 0.8$ A m^{-1}($\omega_H/2\pi \approx 65$ GHz). We observe clear differential magnetoresistance oscillations. At the top of this figure the values of $\epsilon^{\mathrm{dc}} = \omega_H/\omega_c$ are displayed, suggesting an oscillation period $\Delta\epsilon^{\mathrm{dc}} \sim 1$. To confirm these observations, in figure 2(*b*) the Hall-field-induced correction $\Delta r_{xx} = r_{xx} - r_{xx}(J_x = 0)$ is plotted as a function of ϵ^{dc}. Magnetoresistance oscillations are clearly observed up to the seventh order. The first peak appears at $\epsilon^{\mathrm{dc}} \sim 0.95$, for higher ϵ oscillations the maxima occur at $\epsilon^{\mathrm{dc}} \approx j$, with j an integer; while the minima are very close to $\epsilon^{\mathrm{dc}} \approx j + 1/2$. These results are very similar to the experimental findings of Zhang *et al* [24], although the localization of maxima (minima) close to integer (half-integer) is here obtained when $\gamma = 2$, whereas in that work they correspond to the selection $\gamma = 1.9$. The amplitude of the differential resistance oscillations displays a rapid decay as the magnetic field decreases. This decay can be parametrized by the Dingle factor $\delta = \exp(-\pi/\omega_c\tau_s)$.

It is now interesting to consider the magnetoresistance behavior when the system is subject simultaneously to ac (microwave) and dc (Hall) fields. In the experimental work of Zhang

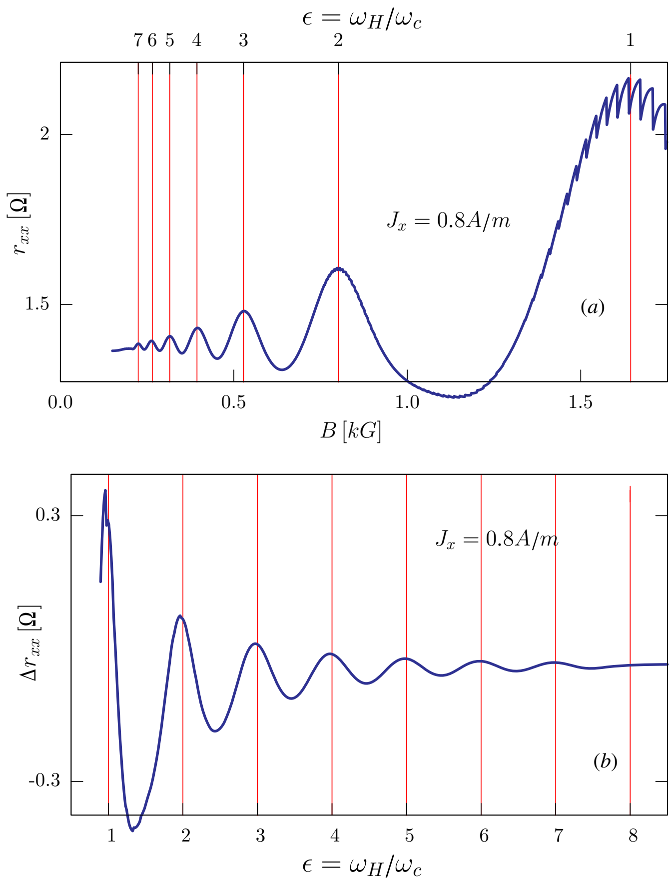

Figure 2. (*a*) Differential resistance $r_{xx}(B)$. (*b*) Correction to differential resistance $\Delta r_{xx} = r_{xx} - r_{xx}(\mathcal{J}_x = 0)$ versus ϵ^{dc}. Here $\mu \approx 2.5 \times 10^7\ \mathrm{cm}^2\ \mathrm{V}^{-1}\ \mathrm{s}^{-1}$, the other parameter are the same as in figure 1.

et al [25] it was found that the dc excitation affects the microwave photoresistance in a nontrivial way. In particular, it was found that maxima (minima) in the differential resistance can evolve into minima (maxima) and back as a result of an interplay of the ac- and dc-induced effects. If MIRO and HIRO behavior are governed by the parameters ϵ^{ac} and ϵ^{dc} respectively, the question arises whether there is a simple parameter that describes resistance oscillations when the system is subject to both ac and dc excitations. Remarkably, it was found that resistance oscillation data display a periodic behavior if plotted as a function of [25]

$$\epsilon^{\mathrm{eff}} = \epsilon^{\mathrm{ac}} + \epsilon^{\mathrm{dc}} = \frac{\omega + \omega_H}{\omega_c}. \tag{27}$$

Similar results arise within the present theoretical formalism. Figure 3 shows the differential resistance r_{xx} as a function of ϵ^{eff}. Results are displayed for three values of the current density: (a) $\mathcal{J}_x = 0$, (b) $\mathcal{J}_x = 0.05\ \mathrm{A\ m}^{-1}$, (c) $\mathcal{J}_x = 0.1\ \mathrm{A\ m}^{-1}$. It is observed that the magnetoresistance oscillations are periodic in ϵ^{eff} with a period $\epsilon^{\mathrm{eff}} \approx 1$. Furthermore, it is observed that the positions of both the maxima and minima of r_{xx} change as $\mathcal{J}$ is increased. For the $\mathcal{J}_x = 0$ the minima and maxima are localized at $\epsilon^{\mathrm{eff}} \approx j + 1/5$ and $\epsilon^{\mathrm{eff}} \approx j - 1/5$ respectively, corresponding to the MIRO case. With increasing $\mathcal{J}_x$ the positions of the first

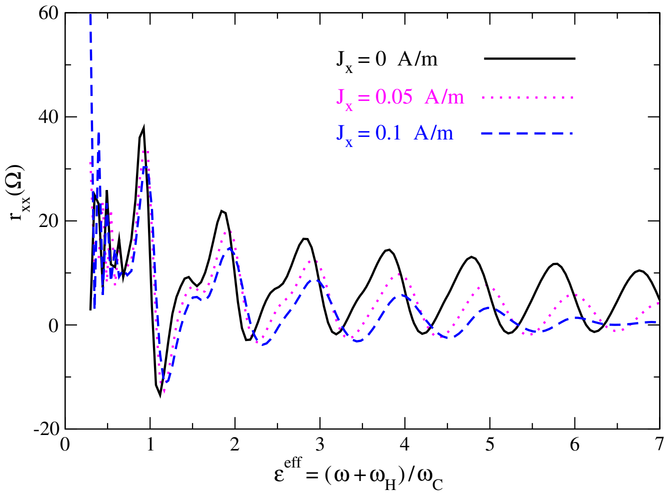

Figure 3. Differential resistance r_{xx} as a function of $\epsilon^{\rm eff} = (\omega + \omega_H)/\omega_c$ obtained under the combined effects of microwave and high density current excitations. The values of the current densities are: $\mathcal{J}_x = 0$, $\mathcal{J}_x = 0.05$ A m^{-1}, and $\mathcal{J}_x = 0.1$ A m^{-1}. The other parameter are the same as in figure 1.

maxima and minima at around $\epsilon^{\rm eff}$ are slightly modified. However as we approach the region of separated Landau levels the positions of the minima and maxima become progressively similar to those corresponding to the HIRO regime, i.e. $\epsilon^{\rm eff} \approx j + 1/2$ and $\epsilon^{\rm eff} \approx j$, respectively. The physical origin to these results can be traced down into the structure of equation (16), observing the arguments of the distribution functions, the electron transitions can be interpreted as a combination of vertical transitions between Landau levels as a result of photon absorption and the tunneling between Hall-field-tilted Landau levels. A more detailed analysis and comparison with the experimental results will be presented elsewhere.

3. Conclusions

We have presented a model to describe the nonlinear magnetoresistance of a 2DEG subjected to microwave irradiation. The method to obtain the exact solution of the time-dependent Schrödinger equation in the presence of arbitrarily strong electric, magnetic and microwave fields exploits the symmetries of the problem. The unitary transformation W is written in terms of the noncommuting guiding center and relative coordinates operators; the corresponding coefficients are obtained from the solutions to the classical equations of motion. Based on this formalism, we provide a Kubo-like formula that takes into account the oscillatory Floquet structure of the problem. We present results for the microwave-induced resistivity oscillations (MIRO) and the Hall-induced resistivity oscillations (HIRO) that are in good agreement with the observations of recent experiments. Hence, we provided a framework to study electron transport properties of a 2DEG in the presence of ac and dc excitations.

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